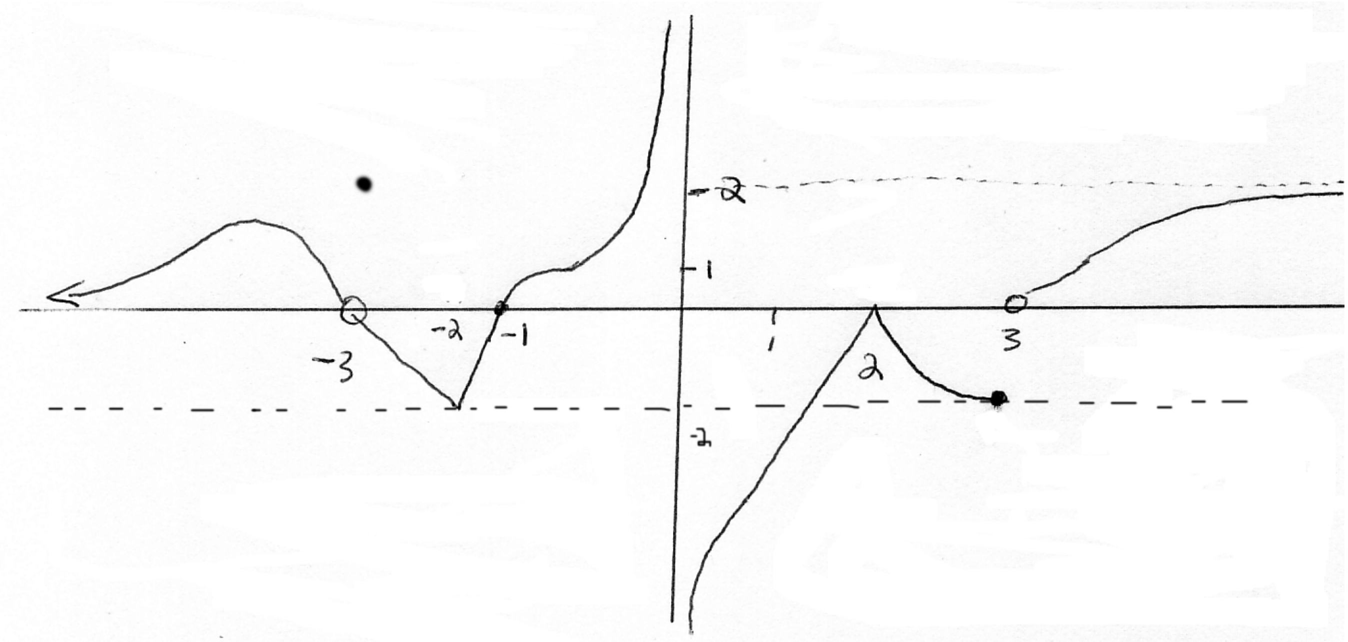
Business Calculus – Sample Exam #1
SHOW ALL WORK, NO CALCULATORS
NO NEED TO SIMPLIFY ANSWERS

For the following five problems, use the function, $f(x)$, graphed below.



1. (5 points) Where does the function fail to be continuous?

Solution: $x = -3, 3.$

2. (5 points) Where does the function fail to be differentiable?

Solution: $x - -3, -2, 2, 3$

3. (5 points) What is $\lim_{x \downarrow 3} f(x)$?

Solution: 0

4. (5 points) What is $\lim_{x \uparrow 3} f(x)$?

Solution: -2

5. (5 points) What is $\lim_{x \to 0} f(x)$?

Solution: $\nexists$

6. (25 points) Find the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$ using the <u>definition</u> of the derivative.

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{1/\sqrt{x+h} - 1/\sqrt{x}}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \\
&= \lim_{h \to 0} \frac{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
&= \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})} = -\frac{1}{2}x^{-3/2}.
\end{aligned}$$

Find the derivative in the following four problems using whatever method you want. Use the fact that $[\tan(x)]' = \sec^2(x)$. Show all work.

7. (15 points) $\left[3x^6 - 6x^4 + 1 + x^{-7}\right]'$.

Solution: $18x^5 - 24x^3 - 7x^{-8}$.

8. (15 points) $\left[(3x^6 + x^2)\tan(x)\right]'$.

Solution: $(18x^5 + 2x)\tan(x) + (3x^6 + x^2)\sec^2(x)$.

9. (20 points) $\left[\dfrac{\tan(x)}{3x^4 + x}\right]'$.

Solution: $\dfrac{(3x^4 + x)\sec^2(x) - (12x^3 + 1)\tan(x)}{(3x^4 + x)^2}$.

10. (20 points) $\left[\tan(x^5 - x^3 + 7)\right]'$.

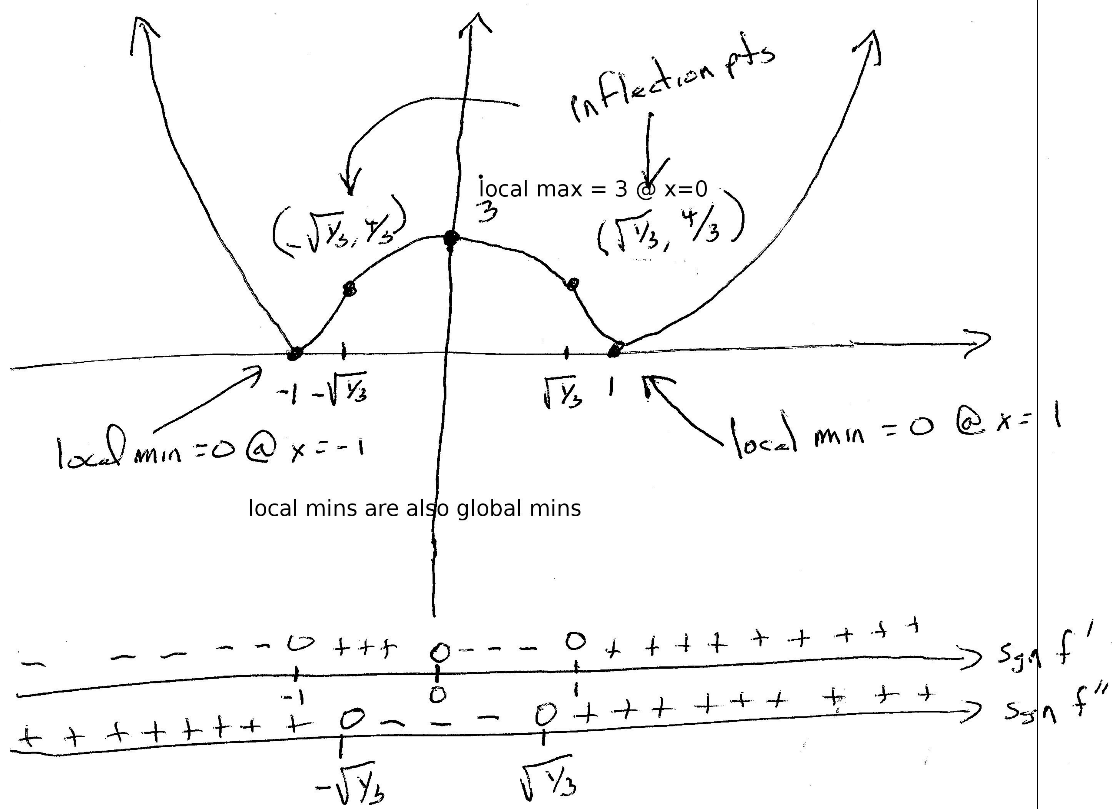
Solution: $[\sec^2(x^5 - x^3 + 7)](5x^4 - 3x^2) = (5x^4 - 3x^2)\sec^2(x^5 - x^3 + 7)$.

11. (25 points) Graph $f(x) = 3x^4 - 6x^2 + 3$ labeling all relative extrema, inflections points, where the function is increasing/decreasing and where function is concave upward/downward. Be sure to identify all of the above!

Solution:

$$\begin{aligned} f'(x) &= 12x^3 - 12x = 12z(x-1)(x+1) \\ f''(x) &= 36x^2 - 12 = 36\left(x - \sqrt{1/3}\right)\left(x + \sqrt{1/3}\right) \\ \infty &= \lim_{x\uparrow\infty} f(x) = \lim_{x\downarrow-\infty} f(x) \end{aligned}$$

Notice that $f(-x) = f(x)$ so graph is symmetric about y–axis.



$f \uparrow$ for $(-1, 0) \cup (1, \infty)$
$f \downarrow$ for $(-\infty, -1) \cup (0, 1)$
f concave up for $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$
f concave down for $(-\sqrt{1/3}, \sqrt{1/3})$.

12. (25 points) Find the slope of the tangent line to $(x^2 + 2y)^3 = 2xy^2 + 64$ at $(0, 2)$. (Page 172, Exercise #26)

Solution:

$$\begin{aligned}(x^2+2y(x))^3 &= 2xy^2(x)+64\\ 3(x^2+2y(x))^2(2x+2y'(x)) &= 2y^2(x)+4xy(x)y'(x)\\ 3(0^2+2(2))^2(2(0)+2y'(0)) &= 2(2^2)+4(0)2y'(0)\\ 96y'(0) &= 8\\ y'(0) &= \frac{1}{12}.\end{aligned}$$

13. (25 points) When electronics store prices a certain brand of stereo at p hundreds dollars per set, it is found that x sets will be sold each month where $x^2+2p^2=41$.

    1. Find the elasticity of demand for the stereos.
    2. For a unit price of $p=4$ (that is \$400) is the demand elastic, inelastic or of unit elasticity?

    (Page 251, Exercise 40)

Solution:

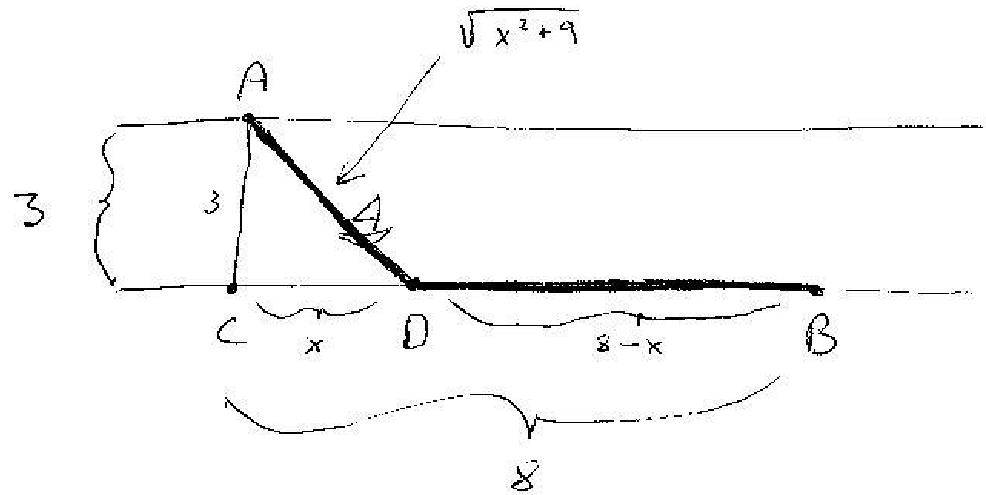
$$\begin{aligned}x^2+2p^2 &= 41\\ x^2 &= 41-2p^2\\ \tilde{D}(p) &= x=\sqrt{41-2p^2}\\ E(p) &= \frac{-p}{\sqrt{41-2p^2}}\frac{1}{2}(41-2p^2)^{-1/2}(-4p)=\frac{2p^2}{41-2p^2}\\ E(4) &= \frac{32}{9}>1.\end{aligned}$$

Elasticity of Demand is elastic when $p=4$.

14. (25 points) A man launches his boat from point A on a bank of a straight river, 3km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible (see figure). He could row his boat directly across the river to point C, then run to B, or he could row directly to B, or he could row to some

point D between C and B and then run to B. If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible. (Assume the speed of the water is negligible compared to the speed the man rows.)

Solution:



$$\begin{aligned}
t(x) &= \frac{\sqrt{3^2+x^2}}{6} + \frac{8-x}{8} \\
0 &= t'(x) = \frac{x}{6\sqrt{9+x^2}} - \frac{1}{8} \\
0 &= \frac{x}{6} - \frac{\sqrt{9+x^2}}{8} \\
\sqrt{9+x^2} &= \frac{4}{3}x \\
9+x^2 &= \frac{16}{9}x^2 \\
x^2 &= \frac{81}{7} \\
x &= \frac{9}{\sqrt{7}} = \frac{9\sqrt{7}}{7}.
\end{aligned}$$